Feasibility of 4D transverse phase space measurement at EMTEX

Kai Hock

University of Liverpool and Cockcroft Institute, UK

GSI, 12-13 September 2013

Overview

A requirement at EMTEX

Problems and suggestions

- 2D beam distribution matrix
- Possible generalisaton to 4D
- 2D phase space measurement
- 4D phase space measurement

Feasibility at EMTEX

Conclusion

EMTEX

1. The EMTEX beamline is part of the Universal Linear Accelerator (UNILAC) at GSI.

2. EMTEX stands for EMittance Transfer Experiment. The aim is to demonstrate emittance transfer, e.g. from vertical to horizontal.

3. Positive deuterium ions are injected into EMTEX. A carbon foil in the beamline strips electrons from the ions.

4. Emittance transfer is achieved by placing the foil at the centre of a solenoid, together with a suitable design of the beamline magnets.

A requirement

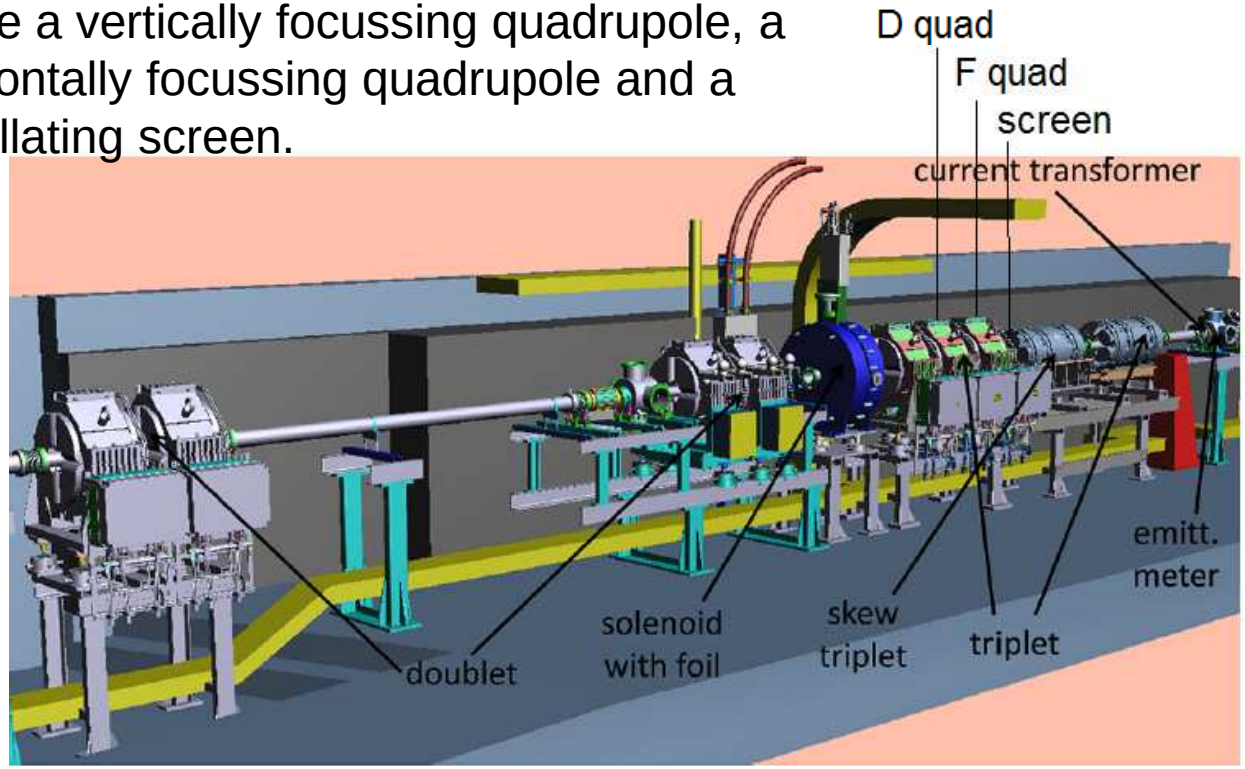
To measure:

1. The four-dimensional symmetric beam matrix

$$C = \begin{bmatrix} \langle xx \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle x'x \rangle & \langle x'x' \rangle & \langle x'y \rangle & \langle x'y' \rangle \\ \langle yx \rangle & \langle yx' \rangle & \langle yy \rangle & \langle yy' \rangle \\ \langle y'x \rangle & \langle y'x' \rangle & \langle y'y \rangle & \langle y'y' \rangle \end{bmatrix}$$

2. The 4D phase space distribution.

Setup: To use a vertically focussing quadrupole, a horizontally focussing quadrupole and a scintillating screen.



EMTEX beamline

For this feasibility study, we assume that there is a screen halfway between the normal and skew triplets.

We assume that the 4D measurement will make use of this screen and the two quadrupoles just before it.

TABLE I. The lattice of the EMTEX beam line.

Element	Effective length [mm]	Gradient [T/m]
Drift	240.5	
Quad	319	+7.276
Drift	203	
Quad	319	−7.726
Drift	4000	
Quad	354	−0.187
Drift	167.5	
Quad	354	+3.287
Drift	500	
Drift	300	
Half sol	100	+1.00 T
Foil	0	22 μg/cm^2, $\Delta\varphi = 0.226$ mrad
Half sol	100	+1.00 T
Drift	300	
Drift	200	
Quad	319	+10.464
Drift	201	
Quad	319	−9.431
Drift	201	
Quad	319	+8.421
Drift	500	
Skew quad	200	+5.110
Drift	20	
Skew quad	400	−2.229
Drift	20	
Skew quad	200	+8.861
Drift	500	
Quad	200	−7.016
Drift	20	
Quad	400	+7.429
Drift	20	
Quad	200	−7.806
Drift	1289	

Vertically focussing

Horizontally focussing

Assume a screen here

Normal triplet

Skew triplet

C. Xiao, et al (2013)

TABLE II. The beam parameters at the entrance and exit of the EMTEX beam line. An initial water-bag distribution was assumed for the tracking simulations.

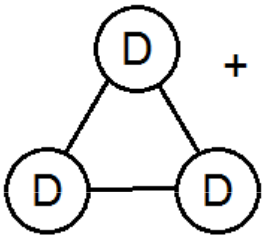
Parameters	Entrance	Exit
α_x/α_y	−1.21/ − 2.28	0.00/0.00
β_x/β_y [mm/mrad]	21.80/15.01	7.18/7.13
$\varepsilon_x/\varepsilon_y$ [mm mrad]	0.509/0.510	0.256/1.144

Foil stripping

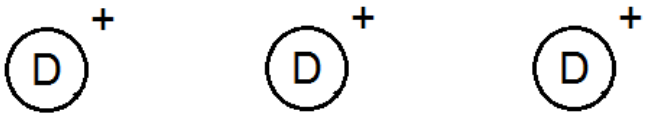
At the foil, charges are stripped from deuterium ions with energy 11.4 MeV/u.

In publications, different notations have been used for these ions:

At injection:



After stripping:



where D is a deuterium atom with 1 proton and 1 neutron.

Problems and suggestions

Problems

1. There is a standard method for measuring 2D beam distribution matrix, but it is not clear how to measure a 4D beam distribution matrix.

2. Recent simulation work has shown that 4D phase space measurement is possible, but this has not yet been demonstrated by experiments.

Suggestions

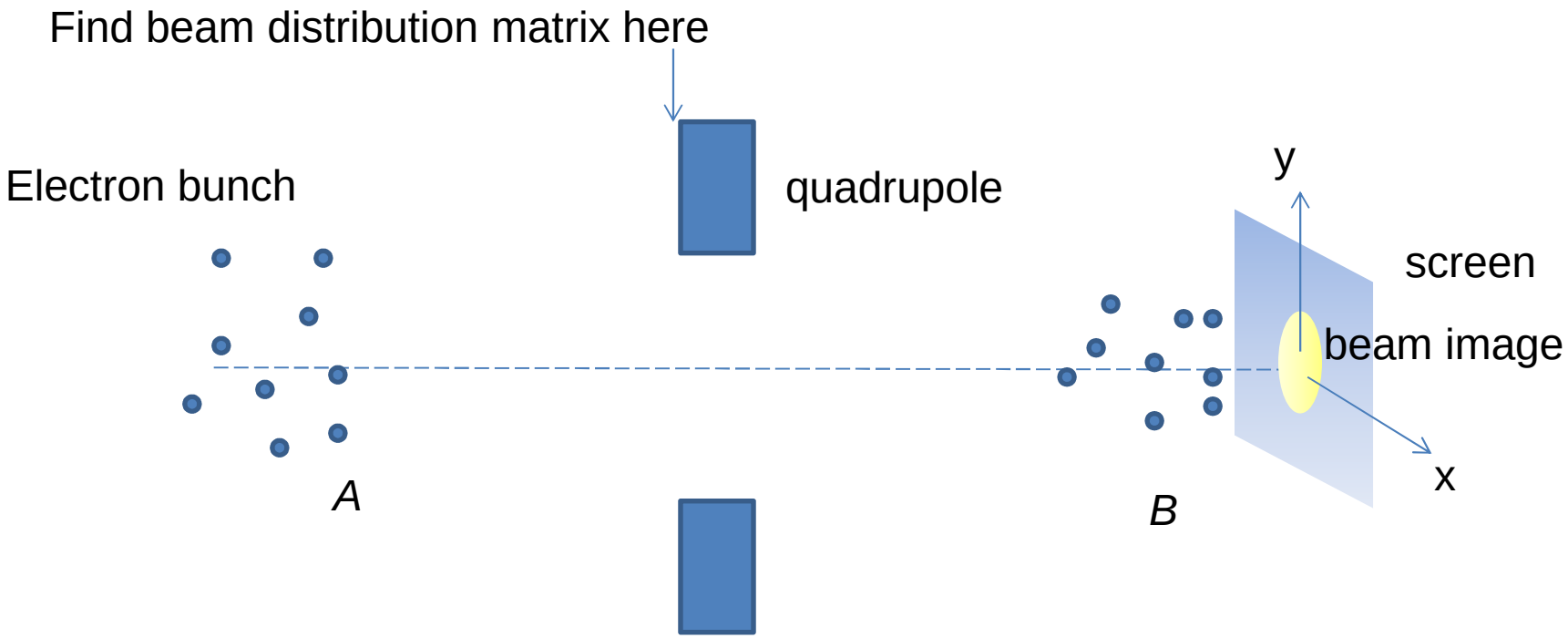
1. Standard 2D beam distribution matrix method may be generalised to 4D. There many unknown parameters to find. We need to check if it is robust.

2. 4D phase space measurement method could be simulated for EMTEX to determine its feasibility. (The 4D beam distribution matrix may also be determined from the phase space distribution.)

2D beam distribution matrix measurement

One way is to use a quadrupole scan setup:

1. Take a photograph of the beam image at B.
2. Change quadrupole strength.
3. Take another photo.
4. Repeat steps 2 and 3 for 100 – 200 times.
5. Then find the beam distribution matrix from the beam images.



Finding the 2D beam distribution matrix

The beam distribution matrix at the entrance to the quadrupole is

$$C = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix}$$

The i^{th} transfer matrix from entrance of quadrupole to screen is

$$M_i = \begin{pmatrix} M_{11,i} & M_{12,i} \\ M_{21,i} & M_{22,i} \end{pmatrix}$$

The beam distribution matrix at the screen is

$$C_i = \begin{pmatrix} \langle x^2 \rangle_i & \langle xx' \rangle_i \\ \langle xx' \rangle_i & \langle x'^2 \rangle_i \end{pmatrix}$$

They are related by

$$C_i = M_i C M_i^T$$

Finding the 2D beam distribution matrix

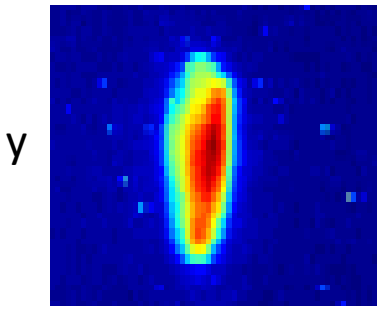
Expand and pick out the $\langle x^2 \rangle$ term:

$$\left\langle x^2 \right\rangle_i = M_{11,i}{}^2 \left\langle x^2 \right\rangle + 2M_{11,i}M_{12,i}\left\langle xx' \right\rangle + M_{12,i}{}^2 \left\langle x'^2 \right\rangle$$

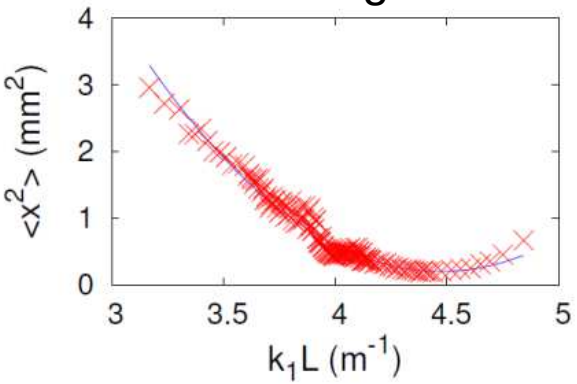
If n images are measured, then there are n such equations. Matrix M can be computed from the quadrupole parameters.

We can then find $\langle x^2 \rangle$, $\langle xx' \rangle$ and $\langle x'^2 \rangle$ that best fit the equations.

beam image



fitting



beam distribution matrix

$$C = \begin{pmatrix} \left\langle x^2 \right\rangle & \left\langle xx' \right\rangle \\ \left\langle xx' \right\rangle & \left\langle x'^2 \right\rangle \end{pmatrix}$$

But how to find 4D beam distribution matrix?

The beam distribution matrix at the entrance to the quadrupole is

$$C = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle & \langle x'y \rangle & \langle x'y' \rangle \\ \langle xy \rangle & \langle x'y \rangle & \langle y^2 \rangle & \langle yy' \rangle \\ \langle xy' \rangle & \langle x'y' \rangle & \langle yy' \rangle & \langle y'^2 \rangle \end{pmatrix}$$

The i^{th} transfer matrix from entrance of quadrupole to screen is

$$M_i = \begin{pmatrix} M_{11,i} & M_{12,i} & 0 & 0 \\ M_{21,i} & M_{22,i} & 0 & 0 \\ 0 & 0 & M_{33,i} & M_{34,i} \\ 0 & 0 & M_{43,i} & M_{44,i} \end{pmatrix}$$

The beam distribution matrix at the screen is

$$C_i = \begin{pmatrix} \langle x^2 \rangle_i & \langle xx' \rangle_i & \langle xy \rangle_i & \langle xy' \rangle_i \\ \langle xx' \rangle_i & \langle x'^2 \rangle_i & \langle x'y \rangle_i & \langle x'y' \rangle_i \\ \langle xy \rangle_i & \langle x'y \rangle_i & \langle y^2 \rangle_i & \langle yy' \rangle_i \\ \langle xy' \rangle_i & \langle x'y' \rangle_i & \langle yy' \rangle_i & \langle y'^2 \rangle_i \end{pmatrix}$$

They are again related by $C_i = M_i C M_i^T$

One possible way

The quantities that can be obtained from the beam images are $\langle x^2 \rangle_i$, $\langle y^2 \rangle_i$ and $\langle xy \rangle_i$.

Try writing down the expanded equations:

$$\left\langle x^2 \right\rangle_i = M_{11,i}{}^2 \left\langle x^2 \right\rangle + 2M_{11,i}M_{12,i}\left\langle xx' \right\rangle + M_{12,i}{}^2 \left\langle x'^2 \right\rangle$$

$$\left\langle xy \right\rangle_i = M_{11,i}M_{33,i}\left\langle xy \right\rangle + M_{12,i}M_{33,i}\left\langle x'y \right\rangle + M_{11,i}M_{34,i}\left\langle xy' \right\rangle + M_{12,i}M_{34,i}\left\langle x'y' \right\rangle$$

$$\left\langle y^2 \right\rangle_i = M_{33,i}{}^2 \left\langle y^2 \right\rangle + 2M_{33,i}M_{34,i}\left\langle yy' \right\rangle + M_{34,i}{}^2 \left\langle y'^2 \right\rangle$$

The right hand sides contain all 10 independent elements in the 4D beam distribution matrix.

The 1st and 3rd equations can be solved separately using the 2D method for $\langle x^2 \rangle$, $\langle xx' \rangle$, $\langle x'^2 \rangle$, $\langle y^2 \rangle$, $\langle yy' \rangle$ and $\langle y'^2 \rangle$.

The 2nd equation contains 4 unknowns $\langle xy \rangle$, $\langle x'y \rangle$, $\langle xy' \rangle$ and $\langle x'y' \rangle$.

In principal, they can be solved by finding a set of values that best fits the above equations, but we should do careful simulation to check if this method is reliable.

Phase Space Measurement

1. The technique described in the previous slide gives the second moments. It is also interesting to look at the phase space.

2. There is a standard method using tomography to measure 2D phase space distribution at a point in a beamline.

3. This method requires a screen to record beam profile, some way to vary the transfer matrix before the screen, and a computer code to process the data.

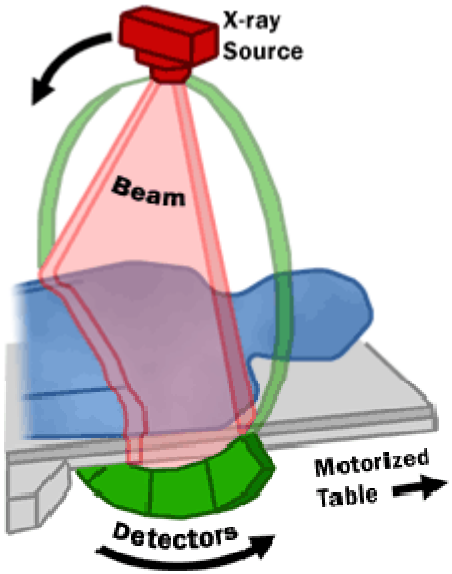
4. The standard method can only work for 2D. Recently, we have developed a method to measure 4D phase space distribution.

5. We have demonstrated using simulation the 4D distribution is possible in the ALICE accelerator at Daresbury using a vertically and a horizontally focussing quadrupole, and a screen.

6. However, the method has not yet been tested in experiments. We shall show by simulation that it is also feasible in EMTEX.

2D phase space measurement ?

In the hospital, tomography is used to image the internal organs. It works by taking X ray projections of the patient from different angles.





http://www.fda.gov/radiation-emittingproducts/radiationemittingproductsandprocedures/medicalimaging/medicalx-rays/ucm115318.htm

If we can take projections of a 2D phase space from different angles, then we can also do tomography on the phase space.

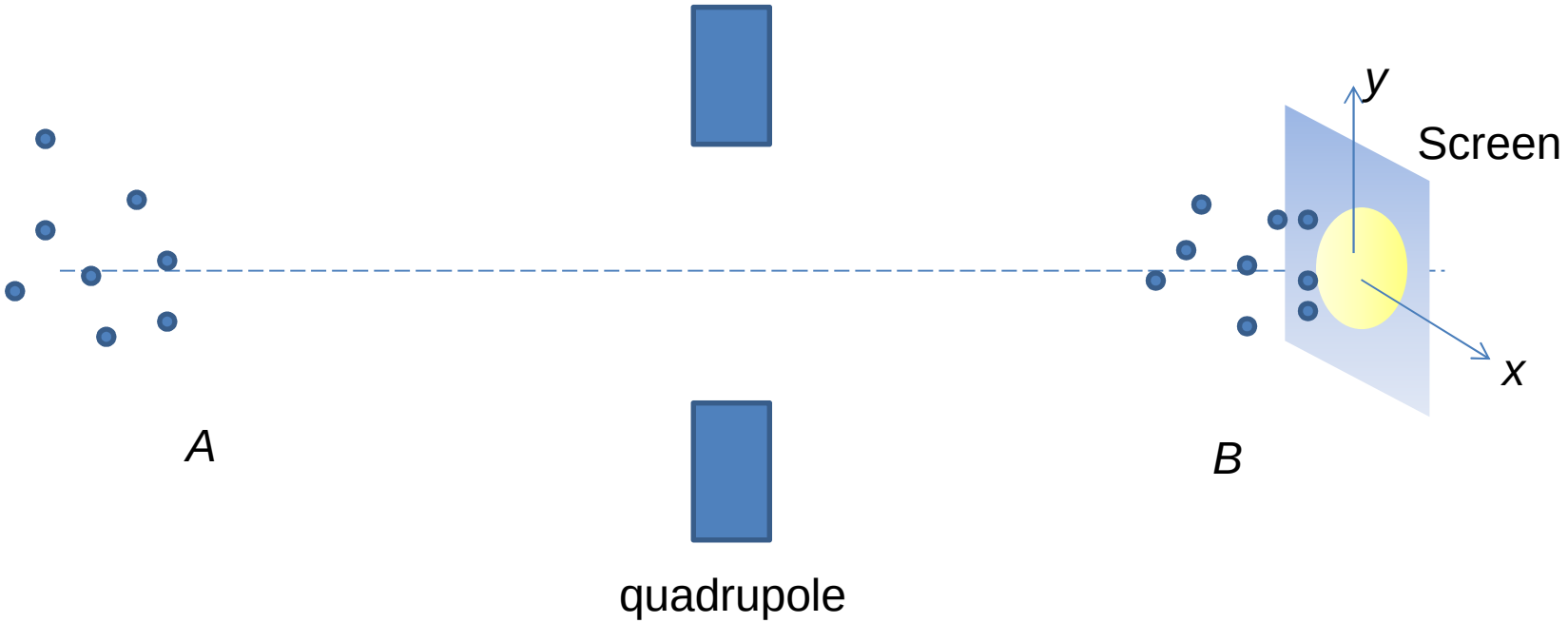
Phase space tomography

Experimental setup

1. We want to find the 2D phase space at a point A in a beamline.

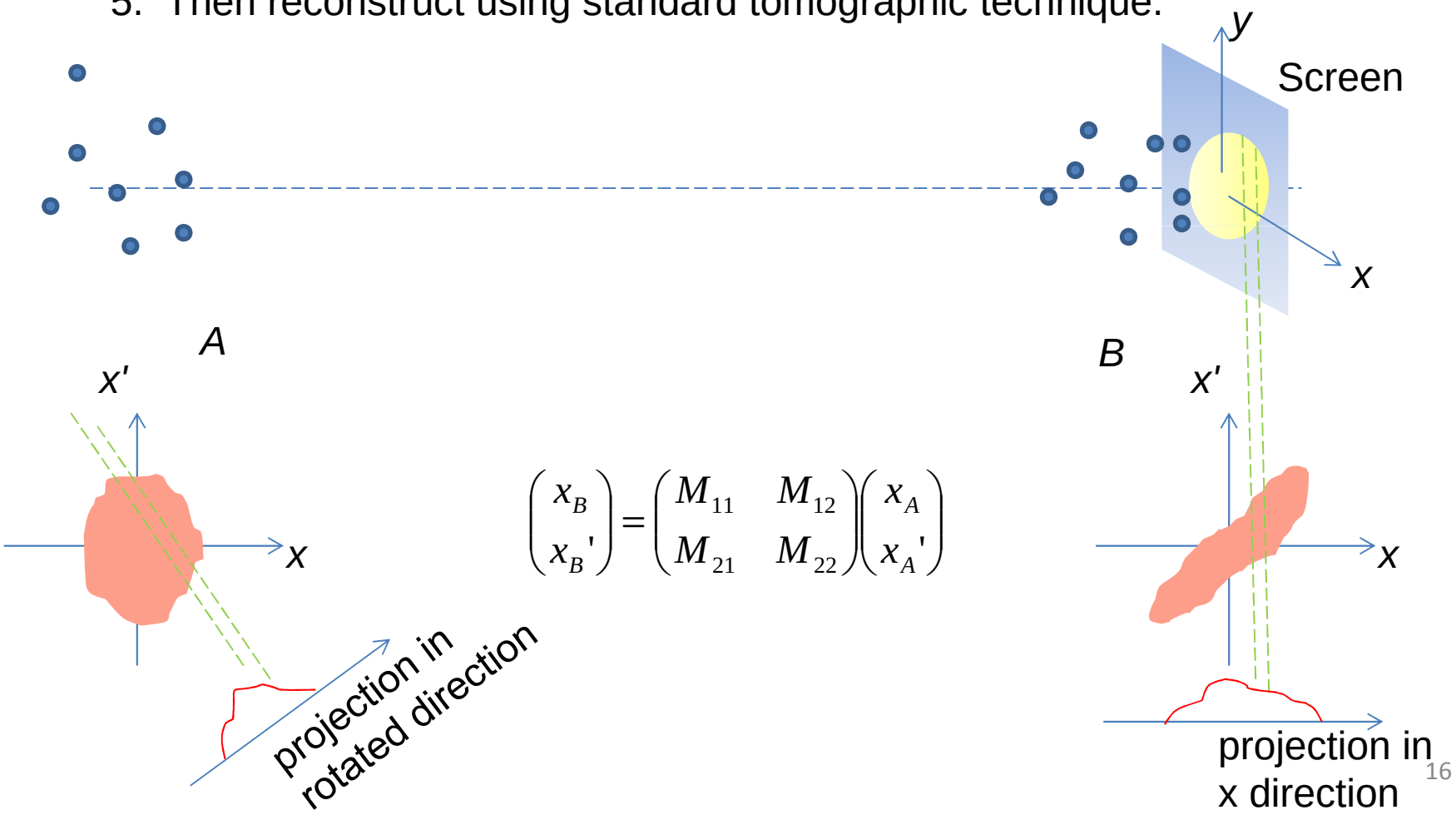
2. The phase space at a screen B can be rotated if we change the transfer matrix between A and B (e.g. by adjusting a quadrupole in between).

3. We can then use a standard method to find the phase space distribution.

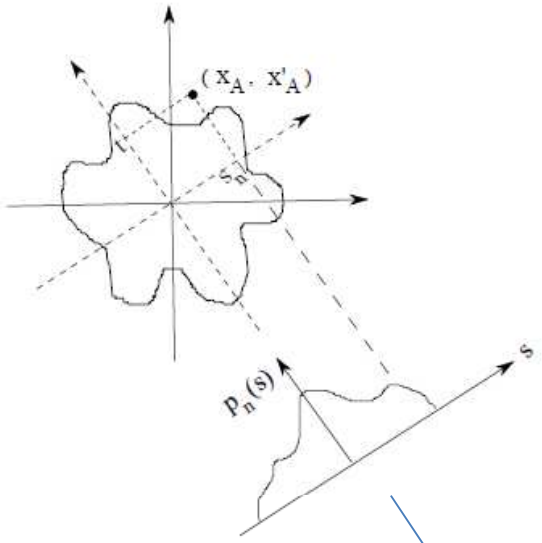


Phase space tomography

Theory: 1. Projection of screen image in x direction = projection of the phase space.
2. This is related to projection in rotated direction at A (reconstruction location).
3. The rotation angle can be obtained from the transfer matrix.
4. By changing the transfer matrix, we can get projections for different angles.
5. Then reconstruct using standard tomographic technique.



Filtered Back Projection (FBP)



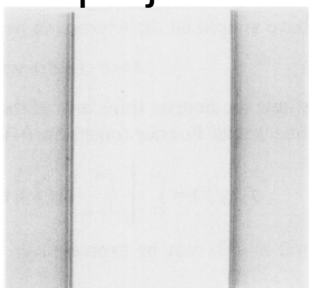
To reconstruct original distribution from projections:

1. Fourier transform each projection, filter, invert transform.
2. Spread over the phase space in perpendicular direction
3. Repeat for each angle and add over previous one.

FFT, filter, invert

1 projection

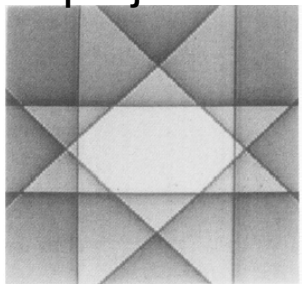
spread



Kak and Slaney (1999)

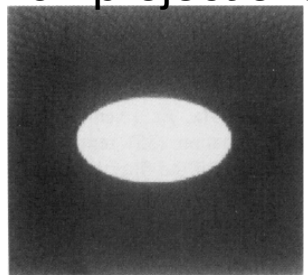
next

4 projections



simulations

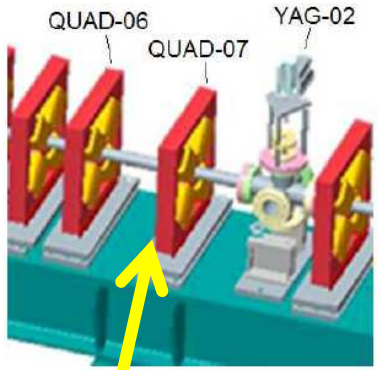
64 projections



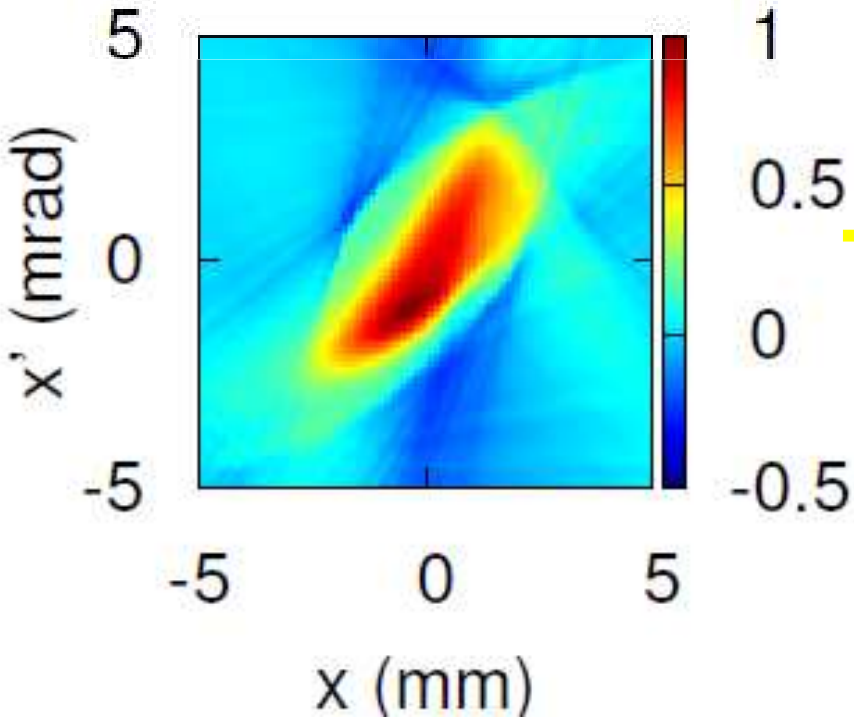
(close to original)

Example: Measurement at Daresbury

ALICE tomography section



1. Take a photograph of the beam image at YAG-02.
2. Change quadrupole strength of QUAD-07.
3. Take another photo.
4. Repeat steps 2 and 3 for 100 – 200 times.
5. Use Filtered Back Projection (FBP) to reconstruct distribution at A.

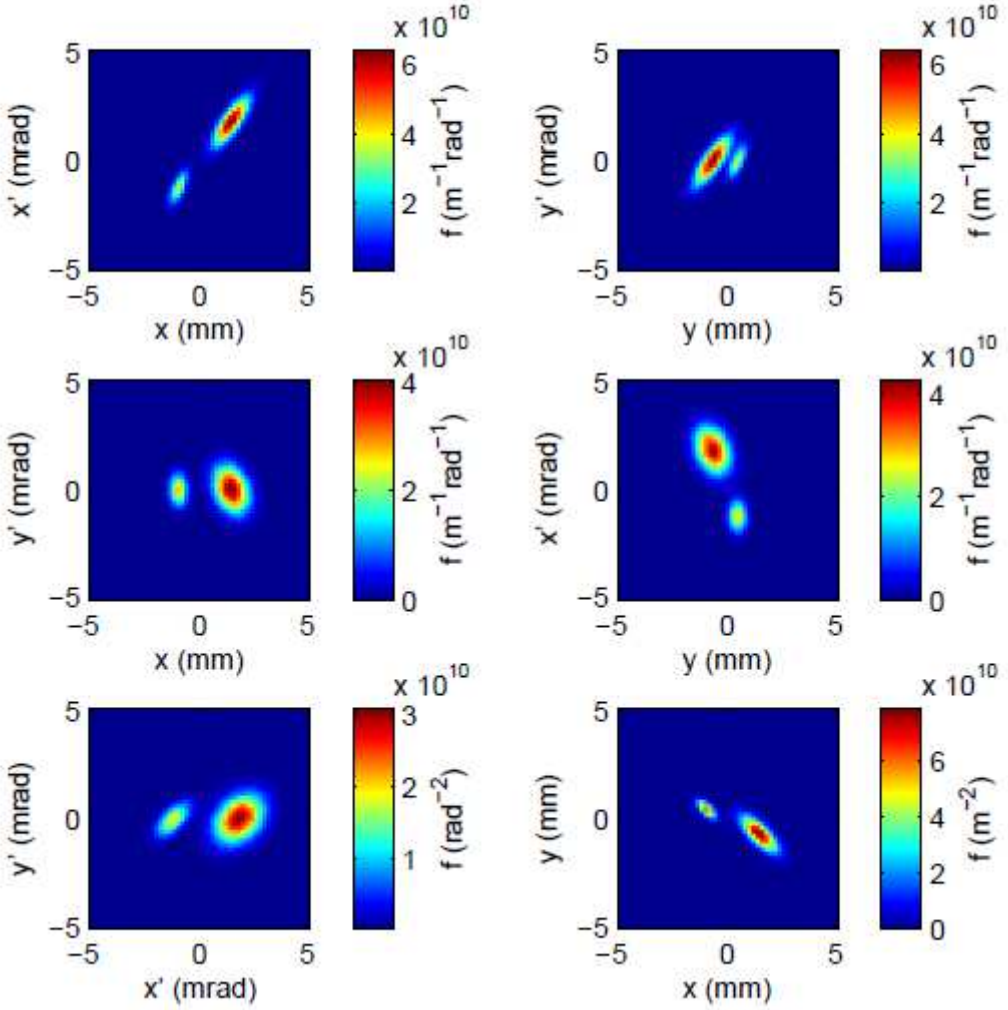


4D phase space measurement

Standard phase space tomography works for 2D phase space (x,x'), (y,y') or (z,δ) only.

Is it possible to measure and reconstruct a full 4D phase space like (x,x', y,y')?

Projections from
a hypothetical 4D
distribution

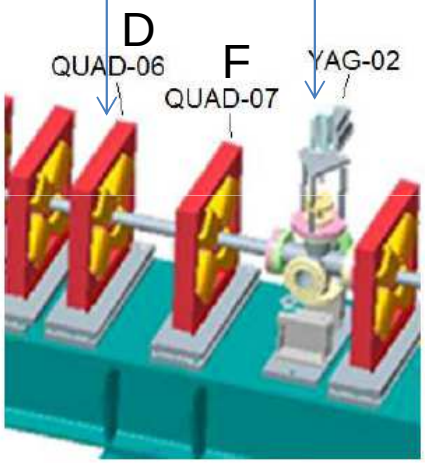


Simulations

4D phase space measurement

Reconstruction location

screen



ALICE tomography section

A possible setup:

1. Suppose that a horizontally focussing quadrupole (F) can rotate in (x,x') and a vertically focussing one (D) in (y,y').
2. For each (y,y') angle, record the images for different (x,x') angles.
3. Repeat this for different (y,y') angles.
4. The full set of recorded images can then be processed using a series of 2D reconstructions to reconstruct the 4D phase space.

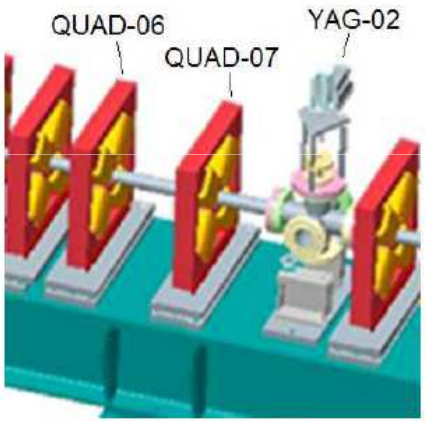
Details, equations and algorithm are given in:

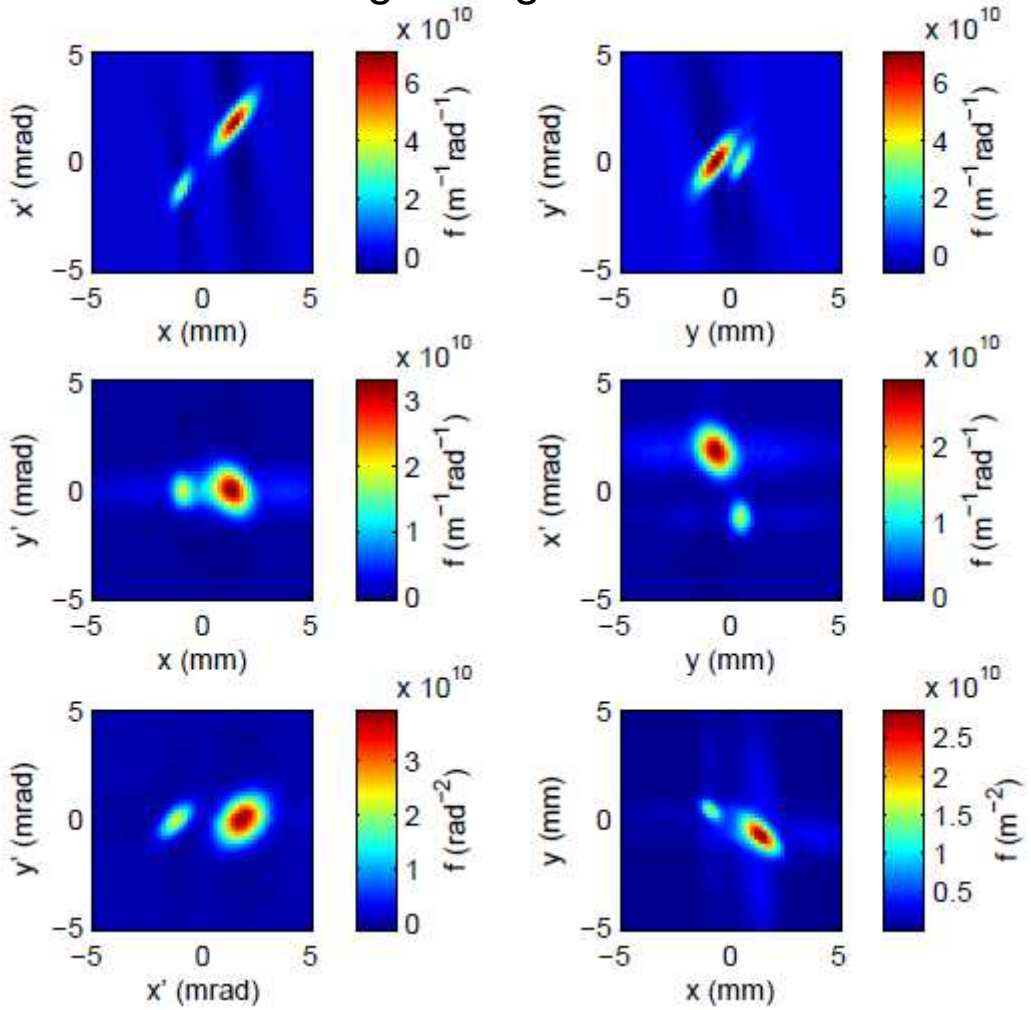
K. M. Hock and A. Wolski. "Tomographic Reconstruction of the Full 4D Transverse Phase Space", submitted to NIMA (2013).

Simulation of a measurement

Using 2 quadrupoles in ALICE, range of angles limited to 150^{o} in both x and y.

Reconstructed 4D distribution still shows good agreement.





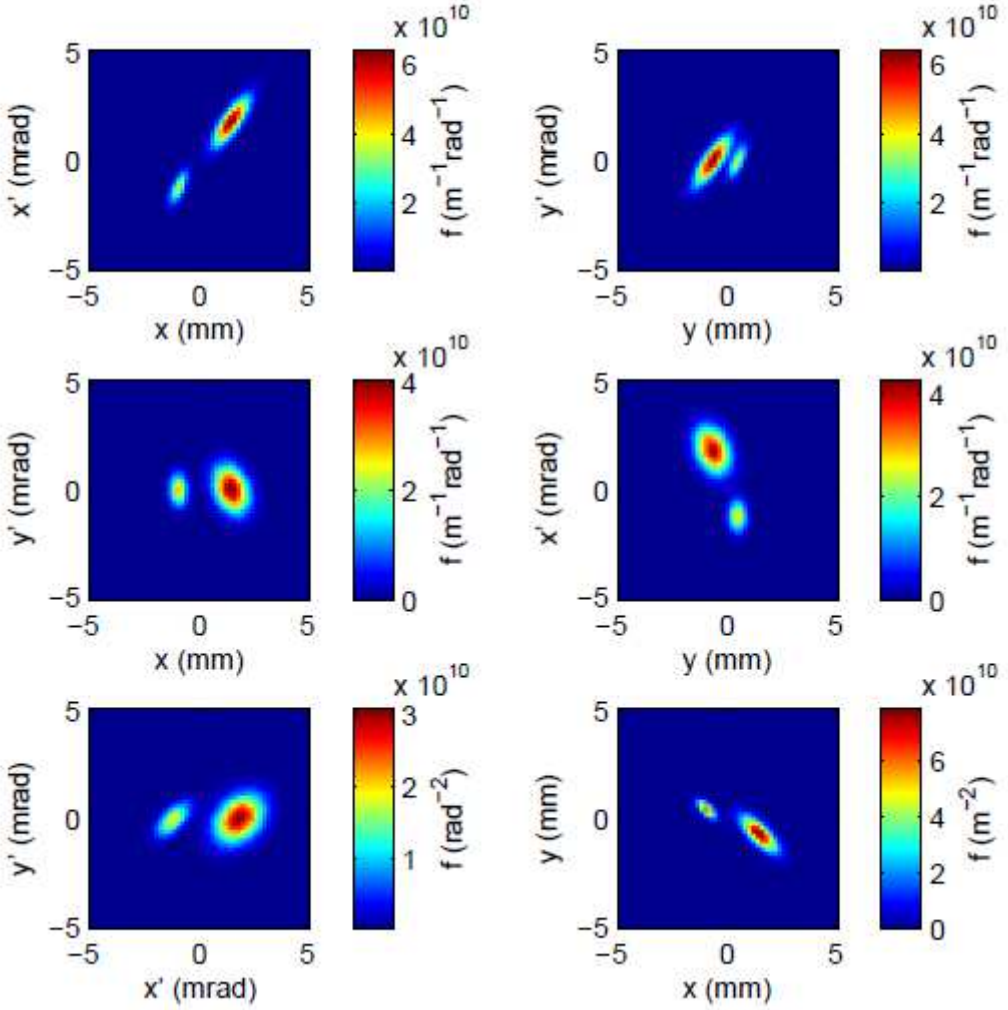
Simulations

4D phase space measurement

Standard phase space tomography works for 2D phase space (x,x'), (y,y') or (z,δ) only.

Is it possible to measure and reconstruct a full 4D phase space like (x,x', y,y')?

Projections from
a hypothetical 4D
distribution

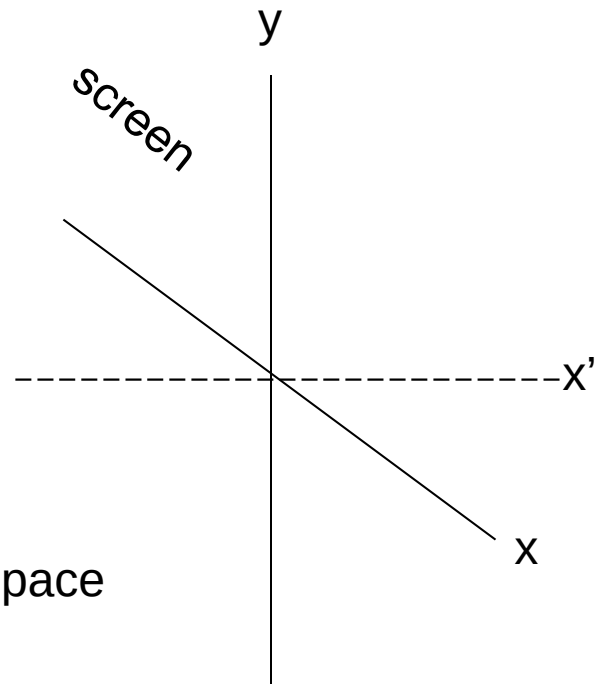


Simulations

4D theory: From 4D to 2D

Define 4D phase space distribution at A (reconstruction location) as number density of electrons per unit phase space volume:

$$f(x, x', y, y')$$



Suppose that this is mapped to B, the screen, as

$$f_B(x, x', y, y')$$

Screen image is projection of 4D phase space to 2D (x,y) plane of screen:

$$I(x, y) = \int \int f_B(x, x', y, y') dx' dy'$$

Think of this in 2 steps: a projection to a 3D (x,x',y) phase space

$$g_y(x, x') = \int f(x, x', y, y') dy'$$

followed by a projection of this 3D phase space to the 2D screen

$$I(x, y) = \int g_y(x, x') dx'$$

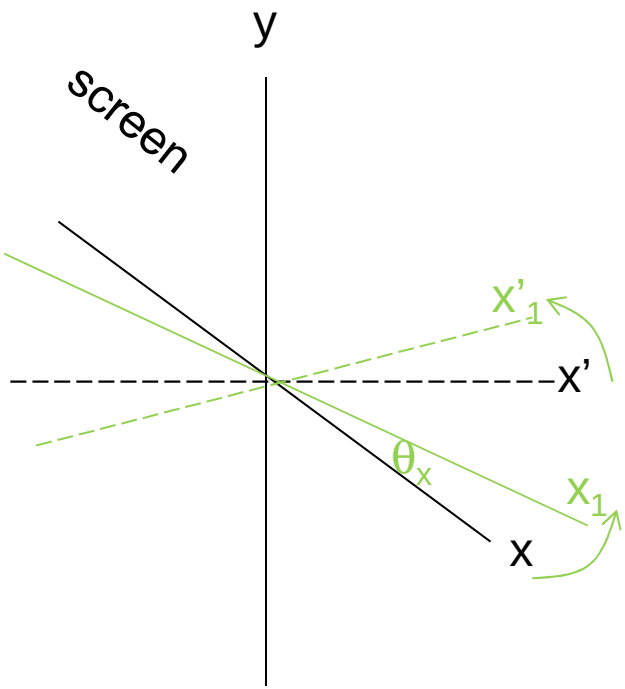
Visualise the 3D phase space with an x' axis perpendicular to the screen.

4D theory: From 2D to 3D

Then we can think of a horizontal line on the screen as a projection from the 3D phase space in the x direction – for a given y.

If we can somehow rotate the (x,x') space about the y axis, we can collect a set of these projections and reconstruct the distribution for each y. Then if we repeat this for every y, we would have reconstructed the (x,x',y) phase space.

From 2D phase space reconstruction, we already know that we can rotate the (x,x') space using a horizontally focussing quadrupole. So we know how to reconstruct (x,x',y).



Recall that this 3D phase space is a projection from the 4D phase space:

$$g_y(x, x') = \int f(x, x', y, y')dy'.$$

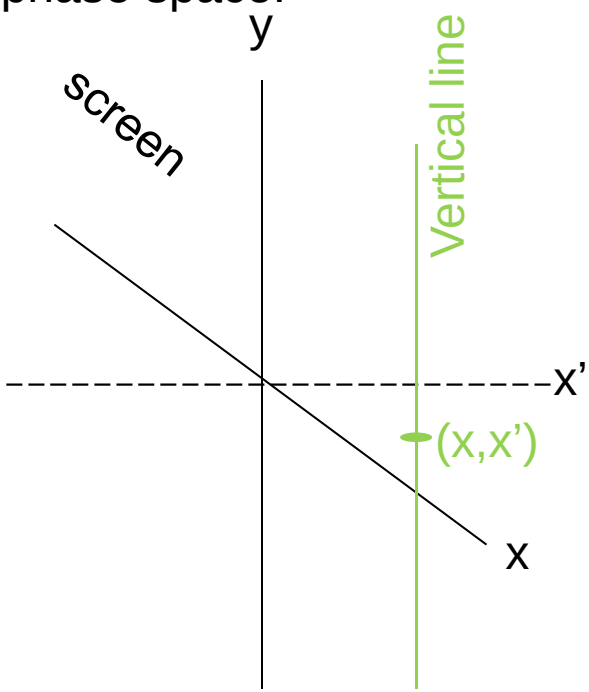
Think of a vertical line in this 3D phase space for a given (x,x'). This is actually a projection from the 4D phase space in the y direction.

4D theory: From 3D to 4D

Since the vertical line through (x,x') is a projection in y direction, we may be able to reconstruct (y,y') for each (x,x') – if we could rotate the (y,y') phase space.

From 2D phase space reconstruction, we know that we can – by using vertically focussing quadrupole.

Since we can reconstruct (y,y') for each (x,x') , the full 4D phase space can now be reconstructed.



Practical issues

The 4D simulation results use 1 horizontally focussing and 1 vertically focussing quadrupole. These give limited ranges of angles of 150^{o} in x and y, resulting in slight distortion and higher background.

Use of more quadrupoles can potentially increase the range of angles to full 180^{o}.

Feasibility of 4D tomography at EMTEX

1. The previous slides show the feasibility of 4D tomography in an electron beam accelerator at Daresbury.

2. Can the optics in the beamline be adjusted to provide a sufficient range of rotation angle?

3. Another questions is the beam size, This expands during vertical focussing. Is the screen big enough?

4. To answer these questions, we simulate the motion of 30000 D_3^+ down the EMTEX beamline.

EMTEX beamline

For this feasibility study, we assume that there is a screen halfway between the normal and skew triplets.

We assume that the 4D measurement will make use of this screen and the two quadrupoles just before it.

TABLE I. The lattice of the EMTEX beam line.

Element	Effective length [mm]	Gradient [T/m]
Drift	240.5	
Quad	319	+7.276
Drift	203	
Quad	319	−7.726
Drift	4000	
Quad	354	−0.187
Drift	167.5	
Quad	354	+3.287
Drift	500	
Drift	300	
Half sol	100	+1.00 T
Foil	0	22 μg/cm^2, $\Delta\varphi = 0.226$ mrad
Half sol	100	+1.00 T
Drift	300	
Drift	200	
Quad	319	+10.464
Drift	201	
Quad	319	−9.431
Drift	201	
Quad	319	+8.421
Drift	500	
Skew quad	200	+5.110
Drift	20	
Skew quad	400	−2.229
Drift	20	
Skew quad	200	+8.861
Drift	500	
Quad	200	−7.016
Drift	20	
Quad	400	+7.429
Drift	20	
Quad	200	−7.806
Drift	1289	

Vertically focussing

Horizontally focussing

Assume a screen here

Normal triplet

Skew triplet

C. Xiao, et al (2013)

TABLE II. The beam parameters at the entrance and exit of the EMTEX beam line. An initial water-bag distribution was assumed for the tracking simulations.

Parameters	Entrance	Exit
α_x/α_y	−1.21/ − 2.28	0.00/0.00
β_x/β_y [mm/mrad]	21.80/15.01	7.18/7.13
$\varepsilon_x/\varepsilon_y$ [mm mrad]	0.509/0.510	0.256/1.144

Simulation method

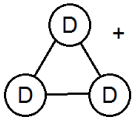
1. Coordinates (x,x',y,y') of a random, 4D Gaussian distribution of 30000 particles is generated, with beam parameters given on the previous slide.

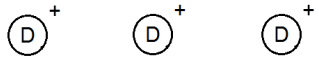
2. Each particle has a mass of $6m_u$, energy of 6x11.4 MeV and charge of +e.

3. It is propagated from start to end of the lattice described on the previous slide.

4. Linear beam dynamic is assumed. So 4x4 transfer matrices are used for each element of the lattice, except for the foil.

5. At the foil:

 (a) every particle 

 becomes 3 particles 

 (b) There is an increase in angular spread of $\Delta\phi$ = 0.226 mrad in both x and y directions. So the standard deviation of x' and y' in the 4D distribution are both increased by $\Delta\phi$;

 (c) Each particle now has a mass of $2m_u$, energy of 2x11.4 MeV and charge of +e.

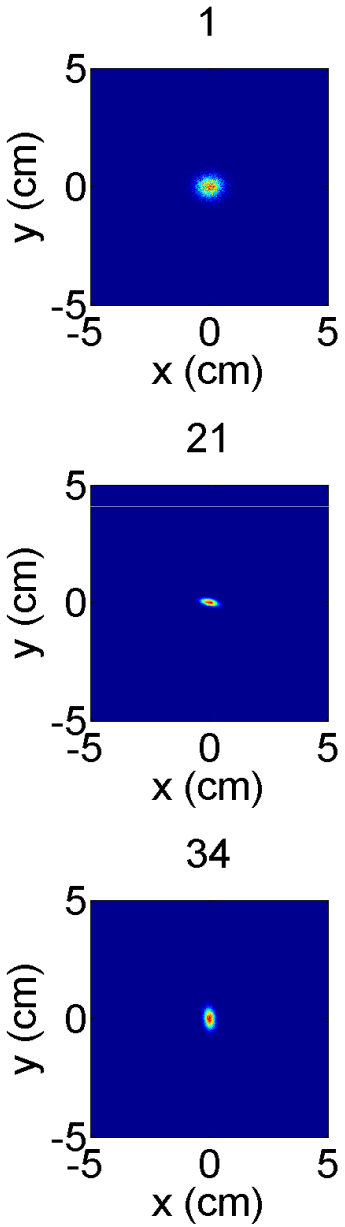
Simulation of beam profiles

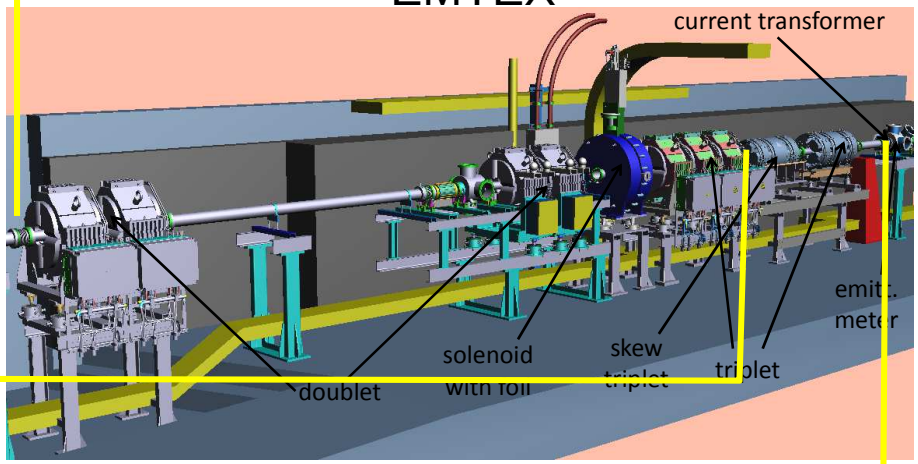
10 cm x 10 cm screens assumed

start

halfway between normal and skew triplets

end





Check simulation against publication

C. Xiao, et al (2013)

TABLE II. The beam parameters at the entrance and exit of the EMTEX beam line. An initial water-bag distribution was assumed for the tracking simulations.

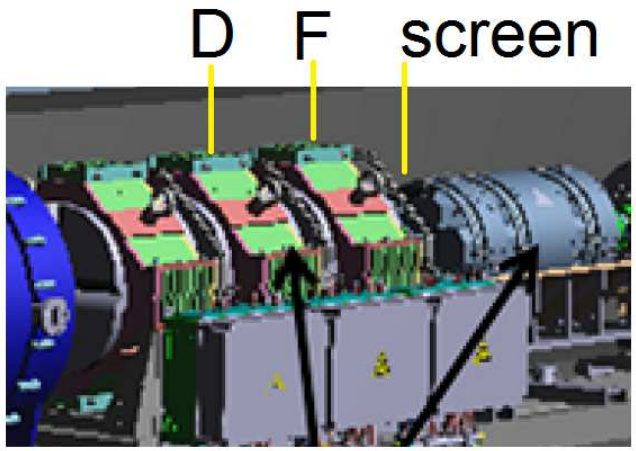
Parameters	Entrance	Exit
α_x/α_y	$-1.21/-2.28$	0.00/0.00
β_x/β_y [mm/mrad]	21.80/15.01	7.18/7.13
$\varepsilon_x/\varepsilon_y$ [mm mrad]	0.509/0.510	0.256/1.144

SIMULATION. Parameters computed from second moments of beam distribution.

Parameters	Entrance	Exit
α_x/α_y	-1.23/-2.28	0.19/0.04
β_x/β_y [m]	22.10/15.02	6.64/6.46
$\varepsilon_x/\varepsilon_y$ [mm.mrad]	0.509/0.509	0.306/1.163

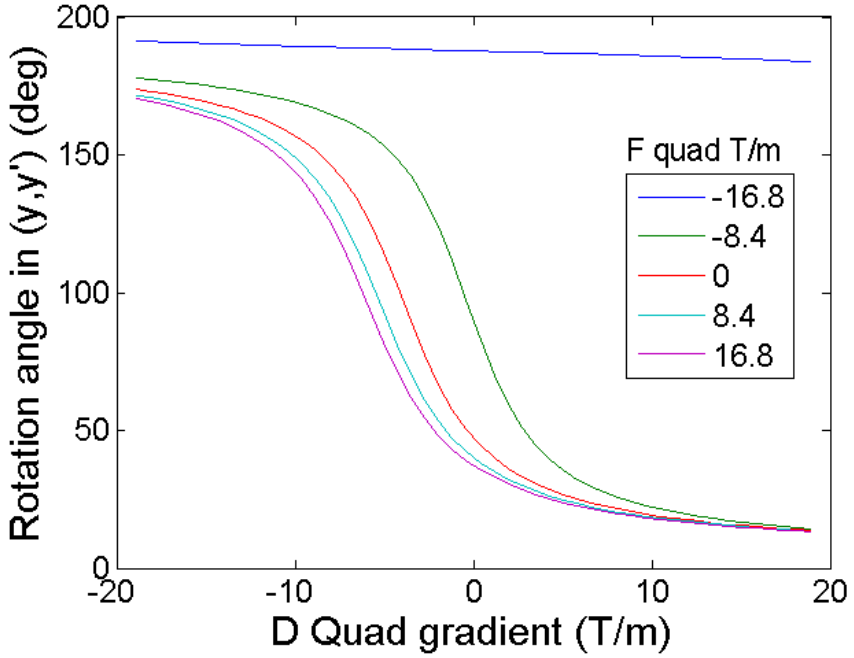
Some differences. Could be due to foil simulation.

Projection angles at entrance face of D quad just before screen

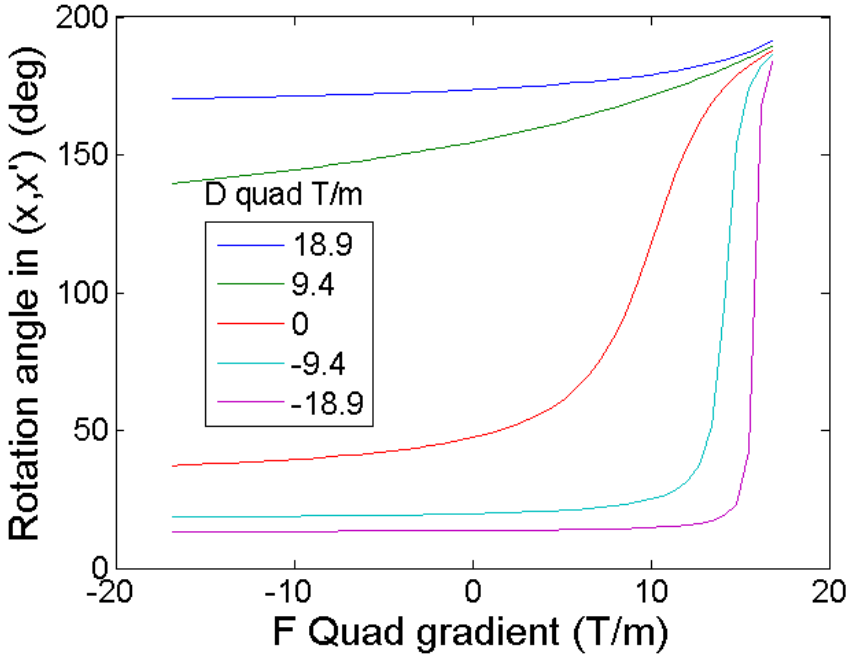


Scan F or D quad just before screen, keeping the strength of the other quad fixed.

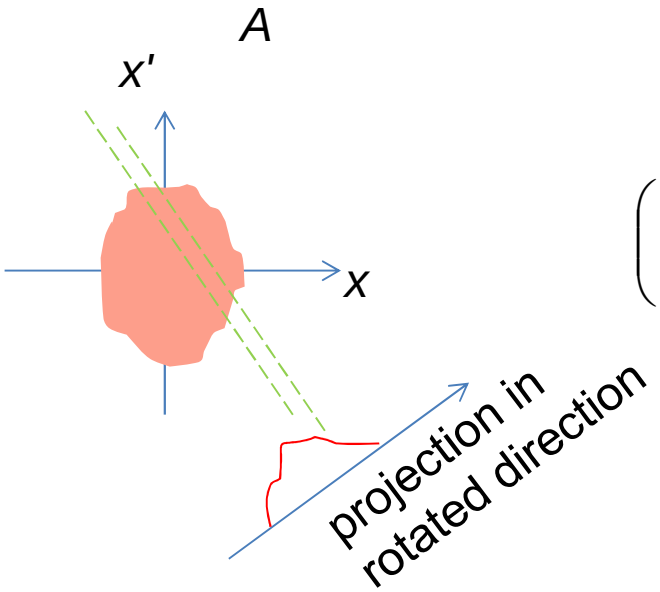
Scan D quad



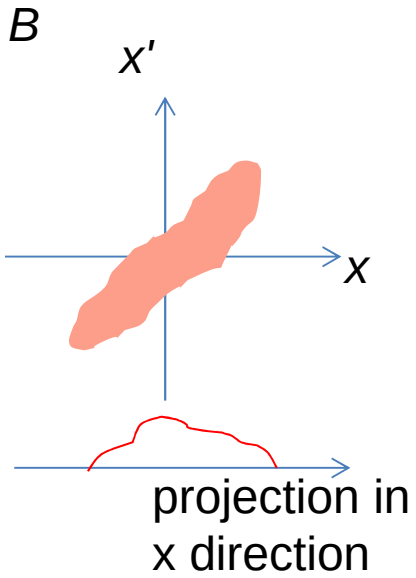
Scan F quad



Rotation Angles



$$\begin{pmatrix} x_B \\ x_B' \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} x_A \\ x_A' \end{pmatrix}$$



1. Rotation angle is determined by transfer matrix.

2. For good reconstruction of phase space, it should cover the full range of 180^{o}.

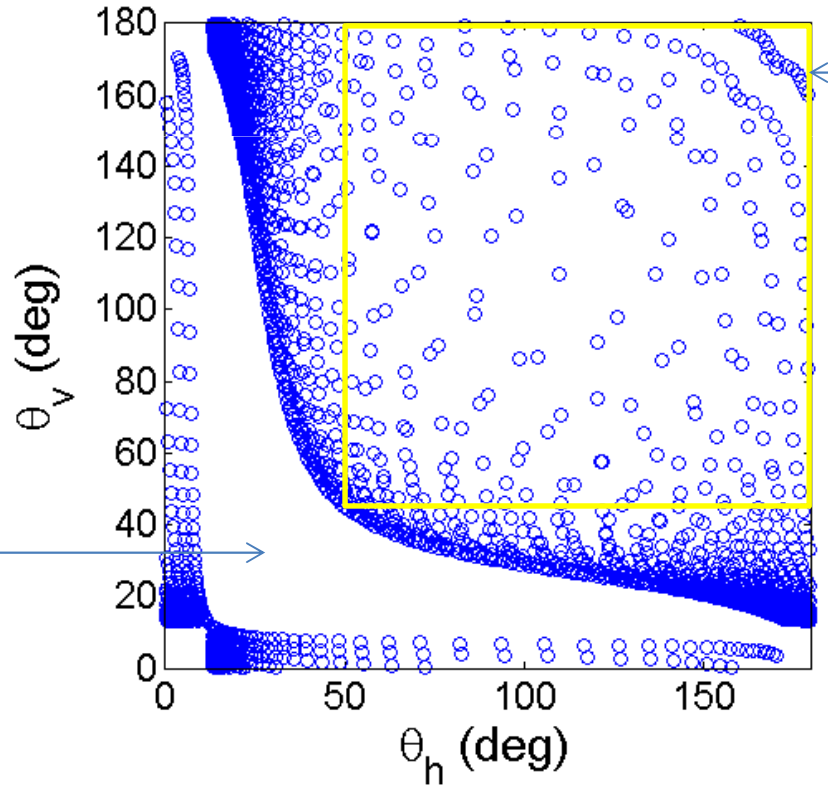
3. For 4D measurement, the full vertical and horizontal range should ideally be accessible for any combination of vertical or horizontal angle.

4. Results in previous slide show that the accessible ranges of angles are reasonably close to 180^{o} for most but not all angle combinations.

Accessible ranges of angles

1. The vertical θ_v and horizontal θ_h phase space rotation angles for possible combinations of F and D quad strengths are calculated and plotted.

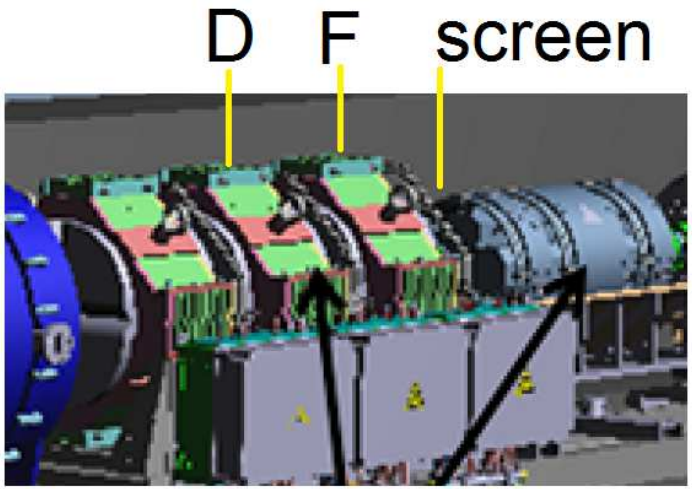
2. The resulting graph shows that at least 150^o x 150^o is accessible. An earlier slide has shown that this is enough for a 4D measurement.



150° x 150°

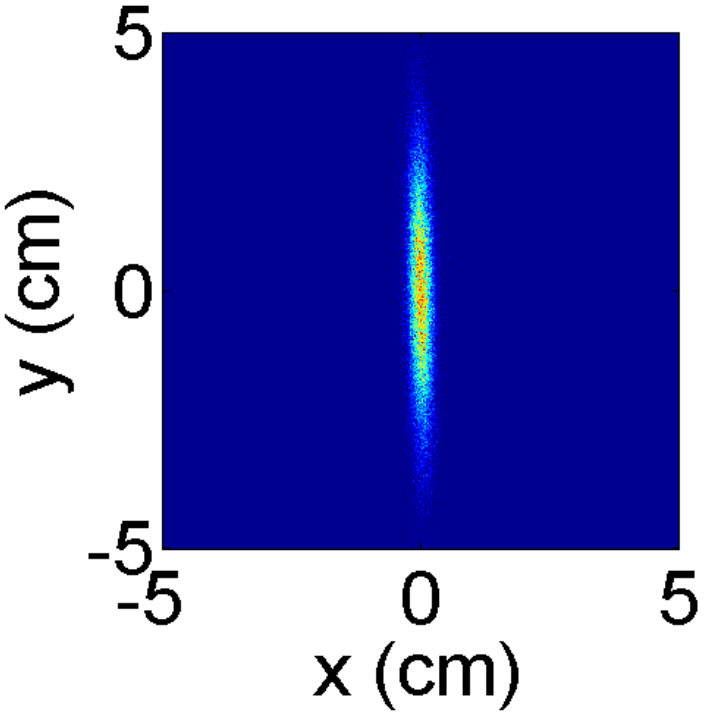
This region is not accessible. It is a limitation of the two quadrupole setup. Using more quadrupoles may help.

Largest beam spots in y and x directions

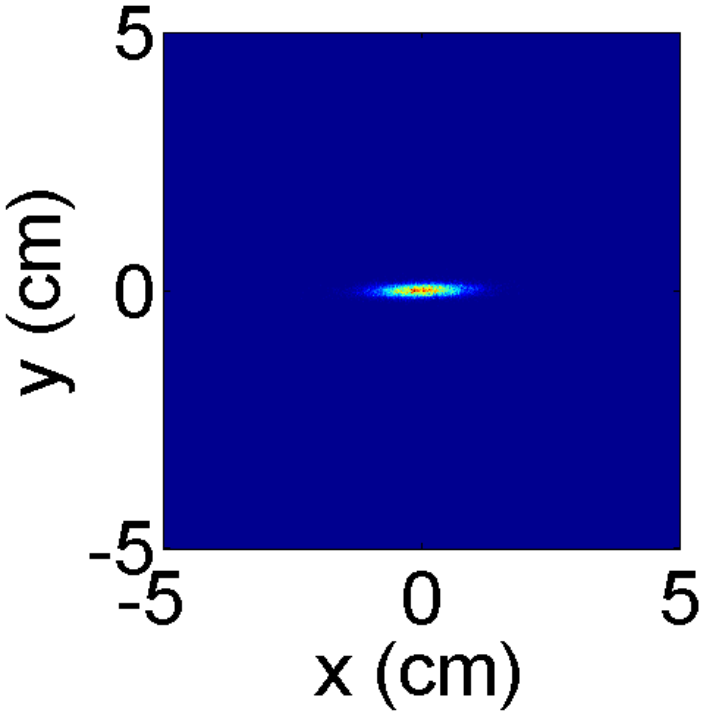


10 cm x 10 cm
screen assumed

D quad 0 T/m, F quad 8.421 T/m



D quad -9.431 T/m, F quad 0 T/m



4D tomography at EMTEX

1. Simulation shows that with the bipolar quadrupoles, the ranges of vertical / horizontal angles cover at least a 150^{o} x 150^{o}, which is sufficient.

2. It is reasonable to think that it is also sufficient for beam distribution matrix measurement since tomography requires more data, but this needs further study.

3. Typical screen size is large enough to capture the defocussed beam images for the assumed starting beam.

4. The above results suggest that 4D phase space measurement at EMTEX is feasible.

Conclusion

1. The requirement to carry out measurements of 4D beam distribution matrix and 4D phase space at EMTEX is described.

2. A standard method for 2D beam distribution matrix measurement is reviewed.

3. A 4D beam distribution matrix measurement is proposed by generalising the 2D method.

4. 2D phase space measurement using tomography is reviewed.

5. Recent development of 4D phase space measurement is reviewed.

6. 4D measurement at EMTEX is studied by simulation and shown to be feasible.

References

C. Xiao, et al, “Single-knob beam line for transverse emittance partitioning”, Physical Review Special Topics – Accelerators and Beams, 16, 044201 (2013).

H. Wiedemann, Particle Accelerator Physics, 3rd ed., Springer-Verlag, Berlin (2007), pp. 164-166.

M. G. Ibison, et al, “ALICE Tomography Section: Measurements and Analysis,” Journal of Instrumentation, 7, P04016 (2012).

K. M. Hock and A. Wolski, “Tomographic Reconstruction of the Full 4D Transverse Phase Space”, submitted to Nuclear Instruments and Methods A (2013).

Thank you